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Aircraft Trajectory Optimization as a Wireless Internet Application

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The problem of finding an optimal aircraft trajectory in close to real time is considered. Using multiple models for different parts of the trajectory, where each model is tailored for the particular conditions, an efficient method has been developed which solves the trajectory optimization problem in a few seconds making interactive use feasible. The different models are used in order to accurately model flight at different conditions such as constant Mach number, constant calibrated airspeed, or constant altitude, using a minimum number of state variables and state equations for each condition. The modeling used and the numerical implementation is described in some detail. A user interface for computing optimal aircraft trajectories using a standard web-browser over the Internet has also been developed. To further illustrate the interactiveness of the developed computational procedures, a mobile Internet implementation has been developed where a cellular phone supporting the Wireless Application Protocol can be used to compute an optimal aircraft trajectory.

Nomenclature

a	Sound speed, m/s
M	Mach number
h	Altitude, m
p	Pressure, Pa
□□	Temperature offset from ISA, K
V	Velocity, m/s
x_E	Distance, km
	Angle of attack, rad
□	Flight path angle, rad
□$_T$	Power level angle (thrust control), rad
□	Density, kg/m^3
q	Dynamic pressure, Pa
m	Aircraft mass, kg
T	Thrust, N
□	Engine installation angle, rad
S_{ref}	Reference area, 16.3 m
D	Drag, N
L	Lift, N
b	Fuel burn, kg/s
C_L	Lift coefficient
C_L^*	Offset parameter
□	Induced drag coefficient
C_{D0}	Zero lift drag coefficient

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C_{Dh} C_{D0} correction for altitude
x Vector of state variables
u Vector of control variables
y Vector of discretized variables

I. Introduction

The use of optimization to find efficient aircraft trajectories is a well established area of research and development. Current methods are based on modeling aircraft performance using Differential Algebraic Equations (DAE) representing the equations of motion and constraints defining limits on the operating conditions of the aircraft. The most commonly used numerical method for discretizing the DAE is to approximate state and control variables as piecewise polynomials and then use collocation to approximately satisfy the equations. Combining the discretized model with nonlinear optimization methods provides an efficient and fairly reliable approach for finding optimal aircraft trajectories. This type of approach was pioneered by Hargraves and Paris[1] and is now in widespread use.[2-5]

Although the current methods for trajectory optimization are becoming quite efficient, interactive use is not often possible. Using an engineering workstation, it is possible to solve fairly large problems in a few minutes. However, in order to reach fully interactive use, computational efficiency has to be improved so that the optimization problem is solved in only a few seconds. Interactive use by nonspecialists also requires that it is simple to state the optimization problem and also simple to interpret and use the results. It is also very important to improve robustness so that solving similar problems takes roughly the same time and that failure to achieve an optimal solution is treated gracefully. In practice it is desirable to use a more crude method to obtain an initial estimate of the solution which is then refined using a more detailed method. By generating a sequence of feasible solutions it is possible to terminate the optimization run in a given time in order to present a result in a few seconds for the cases where the numerical optimization algorithm has difficulties solving a particular case.

The present study extends previous work by the author[5-7] concerning the development of accurate, efficient and reliable methods for trajectory optimization. The focus is here to develop techniques for efficient solution of trajectory optimization problems so that this technique can be used by the pilot as a flight-planning tool before take off. The strategy is to use different aircraft performance models for the different parts of the trajectory improving computational performance without sacrificing modeling accuracy. The different models used are derived and presented together with the numerical treatment used. Finally, the user interface based on Internet and a standard web-browser is presented and illustrated for both static and mobile applications using a cellular phone supporting the Wireless Application Protocol (WAP).[8]

II. Aircraft Performance Modeling

In this study, the trajectory is assumed to be restricted to the vertical plane using altitude and distance from the initial point along some constant heading to define the position of the aircraft. The performance of the aircraft is assumed to be accurately modeled using a point mass approximation. Deriving the equations of motion assuming a point mass model gives the system of ordinary differential equations

$$m\dot{V} = T\cos(\alpha + \varepsilon) - D - mg\sin\gamma \tag{1}$$

$$mV\dot{\gamma} = T\sin(\alpha + \varepsilon) + L - mg\cos\gamma \tag{2}$$

$$\dot{h} = V\sin\gamma \tag{3}$$

$$\dot{m}_f = -b, \tag{4}$$

$$\dot{x}_E = V\cos\gamma \tag{5}$$

where (•) denotes differentiation with respect to time. The mass of the aircraft is split in two parts so that m is the sum of a fixed part and the fuel mass m_f which is treated as a state variable. The angle ε defines the angle of the engine thrust to the fixed body coordinate axis.

The equations of motion defined by Eqs. (1-5) can be rewritten in brief form as

$$\dot{x} = f(x, u) \tag{6}$$

where the vector of state variables is $x = (V, \gamma, h, m_f, x_E)^T$ and the vector of controls $u = (\alpha, \delta_T)^T$. Additional requirements, such as restrictions on Mach number, calibrated airspeed, load factor, and lift coefficient, are implemented as purely algebraic constraints in the form

$$\underline{g} \leq g\,(x, u)\ \leq \overline{g} \tag{7}$$

where $\underline{g}$ and $\overline{g}$ are lower and upper bounds on the algebraic constraints.

The distance state variable x_E does not explicitly depend on the control variables and no other state equation depends on x_E. This makes it possible to compute x_E at any instance by integrating the time histories of V and γ. Consequently, the final distance can be implemented as an algebraic constraint using the expression.

$$x_E(t_F) = \int_{t=0}^{t_F} V\,(t) \cos \gamma(t) dt \tag{8}$$

In the current numerical implementation used for trajectory optimization, the final distance is treated as an algebraic constraint in Eq. (7) eliminating the need to include Eq. (5) in Eq. (6). Consequently, the differential and algebraic constraints are defined so that the state variables are $x = (V, h, m_f, \gamma)^T$.

The aerodynamic lift is modeled as

$$L = q\, S_{ref}\, C_L \tag{9}$$

and the drag as

$$D = qS_{ref}\,(C_{D0} + C_{Dh} + \kappa(C_L - C_L^*)^2) \tag{10}$$

The aerodynamic coefficients of Eqs. (9-10) as well as the engine thrust and fuel burn are all modeled as smooth nonlinear functions with dependencies as listed in Table 1.

Table 1 Dependencies

Coefficient		Dependencies
C_L	=	$C_L(M, \alpha)$
C_L^*	=	$C_L^*(M)$
κ	=	$\kappa(M, C_L)$
C_{Dh}	=	$C_{Dh}(h)$
C_{D0}	=	$C_{D0}(M)$
T	=	$T(M, h, \delta_T, \Delta T)$
b	=	$b(M, h, \delta_T, \Delta T)$

The aerodynamic coefficients and engine data functions are initially available as tabular data. Each tabular function is then modeled as a smooth function using a least-squares fit of B-spline basis functions to the tabular data. The relations between true airspeed, V, Mach number, M, and air density at different altitudes are given through the definition of the standard atmosphere (ISA). The atmospheric model is also defined for arbitrary constant temperature shifts ΔT from the ISA defining the relation between pressure altitude and geopotential altitude.

The intended use of the computed trajectory is to provide a pilot instruction to be presented before take-off. Consequently, it is necessary to present the trajectory in terms of the state variables that are available to the pilot in the aircraft. Further, the trajectory should not be too complicated in order to be useful for the pilot without giving excessive workload.

The most common state variables available to the pilot in the cockpit are calibrated airspeed, Mach number, and pressure altitude. In some cases, flight path angle γ and angle-of-attack are also available. Throttle setting may be defined on a scale but this is not always the case. For example, the throttle control of the Saab 105 (with Swedish Air Force designation SK60) (Ref. 9) does not provide a scale making it difficult to set a desired throttle level.

In order to make the trajectory easy to follow, one would like to present the trajectory in terms of a sequence of segments where some state variable is maintained constant throughout the segment. One example is to present a climb in a sequence as illustrated in Fig. 1.

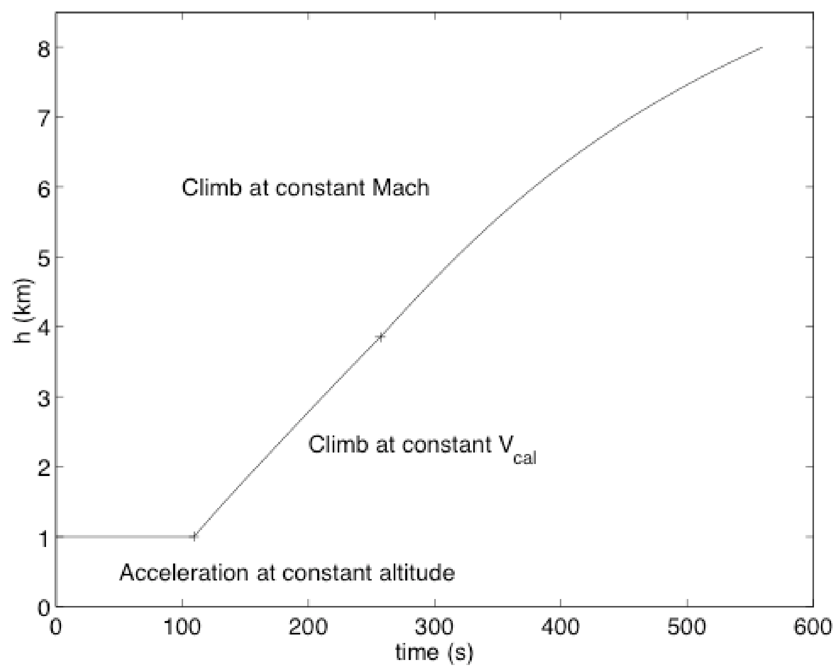

Fig. 1 A standard climb.

First, the pilot is asked to accelerate to a given calibrated airspeed maintaining the altitude constant during the acceleration. Then the pilot initiates a climb maintaining calibrated airspeed constant until a given Mach number is achieved. The final climb to the desired altitude is then performed at constant Mach number. In this case, the optimal control problem is to find the calibrated airspeed and Mach numbers used for the climb. There are consequently only two independent control variables in the problem although the number of state variables and dependent control variables may be quite significant. The climb is assumed to be performed using maximum thrust since this is in most cases optimal for both minimum time and minimum fuel objective functions when considering subsonic aircraft such as the SK60.

For a cruise at constant altitude, a similar simplified optimization formulation is possible. The control variables are here the altitude and Mach number or thrust setting. For some aircraft, such as the SK60, there is no scale on the thrust control apart from flight idle and maximum thrust. Consequently, it is not very useful for the pilot to obtain a suitable thrust setting since it cannot be set in the aircraft. Instead, the optimal Mach number for the cruise is given and the pilot simply adjusts the throttle so that the desired Mach number is achieved at the given altitude.

The original formulation of the equations of motion Eqs. (1-5) does not use the Mach number or calibrated airspeed as state variables. The Mach number and calibrated airspeed are nonlinear functions of airspeed, altitude and atmospheric conditions. Consequently, in order to satisfy a requirement of maintaining constant Mach number or calibrated airspeed it is necessary to impose a large number of nonlinear constraints in the trajectory optimization problem which is very undesirable since the optimization problem becomes larger and significantly more difficult to solve.

To achieve quick and robust solutions to the optimization problem making interactive use possible, it is necessary to proceed differently. In the present study, the trajectory is modeled using different state equations and state variables for the different parts of the trajectory. For example, when maintaining constant calibrated airspeed, the equations are rephrased so that calibrated airspeed is a state variable in place of true airspeed. The significant advantage is that the requirement of constant calibrated airspeed is now a linear constraint in the optimization problem. Furthermore, the model can be derived so that a coarser discretization can be used since it is not necessary to use time steps that are shorter than the period of the phugoid which is necessary considering the original form Eqs. (1-5) of the state equations.[6]

Both these simplifications have significant effect on both efficiency and robustness of the solution of the optimization problem. The different models used in the present study are described in the following section.

III. Use of Multiple Models

The state equations for each model used is derived by defining a suitable set of state variables so that the property that is to be assumed constant appears as a state variable. The corresponding state differential equation is then transformed into an algebraic equation by simply setting the derivative to zero.

In order to derive modified state equations using Mach number or calibrated airspeed as state variables, it is first necessary to review the relation between the new state variables and the original ones used in Eqs. (1-5). Given the true airspeed V, the Mach number is simply obtained using

$$M = V / a_{\cdot}, \tag{11}$$

where $a_{\cdot}$ is the speed of sound which is a function of the altitude h for the International Standard Atmosphere (ISA).

The process of computing the calibrated airspeed is somewhat more involved.[10] Given the true Mach number, the total pressure is obtained as

$$p_{tot} = p\,(1 + \frac{\gamma - 1}{2} M^2)^{\gamma/(\gamma-1)} \tag{12}$$

where p is the static pressure at altitude h from the ISA and $\gamma = 1.4$ for air. The dynamic pressure sensed by the aircraft pitot tube system is thus

$$q_c = p_{tot} - p \tag{13}$$

The indicated Mach number is now defined by

$$\sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{q_c}{p_0} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]} \tag{14}$$

where p_0 is the static pressure at sea-level given by ISA. Finally, the calibrated airspeed is given by

$$V_{cal} = M_i a_0 \tag{15}$$

where a_0 is the speed of sound at sea-level in ISA 0 conditions.

A. Model for constant calibrated airspeed

Rewriting Eqs. (1-5) using calibrated airspeed as a state variable, an expression is required for the time derivative of the true airspeed in terms of the calibrated airspeed and the altitude and the corresponding time derivatives. Differentiating the true airspeed with respect to time gives

$$\dot{V} = \frac{\partial V}{\partial V_{cal}} \dot{V}_{cal} + \frac{\partial V}{\partial h} \dot{h} \tag{16}$$

which, using Eq. (3) and that $\dot{V}_{cal} = 0$, reduces to

$$\dot{V} = \frac{\partial V}{\partial h} V \sin\gamma \tag{17}$$

where V is defined as a function dependent on V_{cal} and h through Eqs. (11-15).

Consequently, a climb at constant calibrated airspeed is accurately modeled by the state equations

$$\dot{h} = V \sin\gamma \tag{18}$$

$$\dot{m}_f = -b \tag{19}$$

and the algebraic equations

$$0 = T\cos(\alpha + \varepsilon) + L - mg\cos\gamma \tag{20}$$

$$0 = T\cos(\alpha + \varepsilon) - D - mg\sin\gamma - m\frac{dV}{dh}V\sin\gamma \tag{21}$$

The only approximation involved in deriving Eqs. (18-21) from Eqs. (1-4) for constant calibrated airspeed is that the time derivative $\dot{\gamma}$ is assumed to be zero which is a valid approximation since γ changes very slowly when calibrated airspeed is constant. The DAE model is now in the form Eqs. (6-7) with $x = (h, m_f)$ and $u = (\alpha, V_{cal}, \gamma, \delta_T)$. Additional algebraic constraints on C_L and M are also included in Eq. (7).

B. Model for constant Mach number

A similar simplified model for climb or descent at constant Mach number is obtained by differentiating true airspeed

$$V = Ma(h) \tag{22}$$

as a function of Mach number and altitude giving

$$\dot{V} = \dot{M}a + M\frac{da}{dh}\dot{h} \tag{23}$$

This expression can, using Eq. (3) and $\dot{M} = 0$, be rearranged as

$$\dot{V} = M^2 a \frac{da}{dh}\sin\gamma \tag{24}$$

Rearranging Eqs. (1-4) for constant M and assuming $\dot{\gamma}$ zero leads to the DAE system

$$\dot{h} = V \sin\gamma \tag{25}$$

$$\dot{m}_f = -b \tag{26}$$

$$0 = T\sin(\alpha + \varepsilon) + L - mg\cos\gamma \tag{27}$$

$$0 = T\cos(\alpha + \varepsilon) - D - mg\sin\gamma - m\frac{da}{dh}a M^2\sin\gamma \tag{28}$$

which is on the form Eqs. (6-7) with $x = (h, m_f)$ and $u = (\alpha, M, \gamma, \delta_T)$. Additional algebraic constraints are considered on C_L and V_{cal}.

C. Constant altitude

For constant altitude using M as state variable, the time derivative of true airspeed is simply given by

$$\dot{V} = a\ \dot{M} \tag{29}$$

which leads to the DAE system

$$ma\ \dot{M} = T\cos(\alpha+\varepsilon) - D \tag{30}$$

$$\dot{m}_f = -b \tag{31}$$

$$0 = T\sin(\alpha+\varepsilon) + L - mg \tag{32}$$

Note that Eq. (3) is eliminated since the altitude is constant. The DAE system Eqs. (6-7) is defined with $x = (M, m_f)$ and $u = (\alpha, h, \gamma, \delta_T)$ with additional algebraic constraints on C_L and V_{cal}. Similar expression can also be derived for constant altitude using the calibrated airspeed as state variable.

D. Discretization

The differential and algebraic equations (DAE) defined by Eqs. (6-7) using the various forms of state variables and state equations discussed in the previous section are discretized using Hermite-Simpson collocation.[11] The state variables x are approximated as piecewise cubic polynomial functions of nondimensional time on each stage. The control variables u are approximated as piecewise linear functions of time. The differential equations are satisfied in integral sense on each time step. The algebraic constraints are enforced both in integral sense on each time step but also at the end points of each time step.

The discretization process transforms the differential equations to algebraic equations dependent on the state and control variables at selected discretization nodal time points. All the discretized state and control variables are stored in the finite dimensional vector y together with the time variables t_i defining change of state model. The differential and algebraic equations can now be stated as a set of purely algebraic equations dependent on the vector y as

$$\underline{l}_i \le c_i(y) \le \overline{u}_i, \qquad i = 1,\ldots,n_c. \tag{33}$$

where $\underline{l}$ and $\underline{u}$ define lower and upper bounds on the nonlinear constraints. Equality constraints are enforced by setting the lower and upper bounds equal.

The different stages based on different sets of state variables must be connected by proper interface conditions to ensure continuity of the flight path. For example, a climb at constant calibrated airspeed which is followed by a climb at constant Mach number must have continuity in the most important variables. A nonlinear condition, on the form Eq. (33), is thus needed to ensure continuity of airspeed. In the present study, the Mach number is used as common definition of airspeed at the interface. The Mach number is a nonlinear function of the calibrated airspeed which is the airspeed state variable in Eqs. (18-21) making the interface condition with the model Eqs. (25-28) having Mach number as airspeed variable nonlinear. However, when connecting a climb at constant Mach number Eqs. (25-28) with a model for constant altitude using Mach number as airspeed state variable, the continuity condition is of course linear.

Continuity conditions are also enforced on altitude h and fuel mass m_f which are defined using a linear function since these variables appear in all the different models used. Requiring continuity in flight path angle and angle of attack is more questionable. If the flight path angle is required to be continuous in the interface between a climb and flight at constant altitude, it is necessary to quite accurately discretize the state variables in the neighborhood of the interface giving a significant increase in the number of variables in y. However, this refined modeling at the interface has minimum influence on the general solution to the trajectory optimization problem in most cases and should be avoided to improve computational performance. Consequently, flight path angle, angle of attack, and thrust setting are all allowed to change discontinuously at model interfaces.

E. Optimization

Defining an objective function such as minimum fuel or minimum time used for a given mission, it is now possible to formulate the optimization problem as

$$\min_{y} f_0(y) \tag{34}$$

$$\underline{l} \le \begin{pmatrix} y \\ C y \\ c(y) \end{pmatrix} \le \overline{u} \tag{35}$$

where f_0 denotes the objective function and the matrix C in Eq. (35) define the linear continuity constraints. The vectors containing all of the constraint lower and upper bounds are denoted $\underline{l}$ and $\underline{u}$ respectively. The simple bound constraints are used to enforce path constraints on the state and control variables.

Current options for the choice of objective function includes total flight time ($\sum t_i$), fuel mass, or distance. Switching from minimization to maximization is simply done by changing the sign of the objective.

The optimization problem Eqs. (34-35) can be quite large involving several hundred and sometimes many thousands of variables and constraints. Numerical efficiency and robustness is most important in order to achieve quick response making interactive use possible. In this project, SNOPT[12] is used to solve Eqs. (34-35) since it has proven efficient for the present application not only by the author[5,13] but also by others.[4] SNOPT uses a sequential quadratic programming (SQP) approach with Quasi-Newton approximations to the required second derivative information. SNOPT efficiently exploits sparsity in the constraint Jacobian when solving the quadratic programming subproblem which is one of the main benefits of the method. However, there are alternatives to using SNOPT, for example, the Boeing developed SOCS system[2] could be used.

To further improve efficiency, multigrid acceleration is used where the problem is first solved on a coarse grid, followed by solves on refined grids using the solution from the previous grid as initial approximation to the solution.

IV. An Internet Application

Apart from making sure that the trajectory optimization problem can be efficiently solved, it is also very important to provide the pilot with a simple and easy to use interface. Today, the by far most widespread interactive user interface is the concept of a web-browser as implemented in different ways under various brand names. The browser interface provides the pilot with a familiar user interface that is easy to use with little or no training. There are very modest requirements on the end user hardware, essentially only a simple PC with some form of Internet access. Recently, Internet access is also possible using cellular phones support the Wireless Application Protocol (WAP).[8]

The current implementation of a system for computing optimal trajectories for the light trainer Saab 105 (SK60) is implemented as shown in Fig. 2. The trajectory optimization system is implemented in Fortran 77 running on a Linux server at KTH which is shown as the application server in Fig. 2. A reasonable Pentium based server provides response times of typically 5-10 s which is sufficiently fast for interactive use. The user can access the application either through a standard Internet interface on a stationary client computer or using a WAP capable cellular phone.

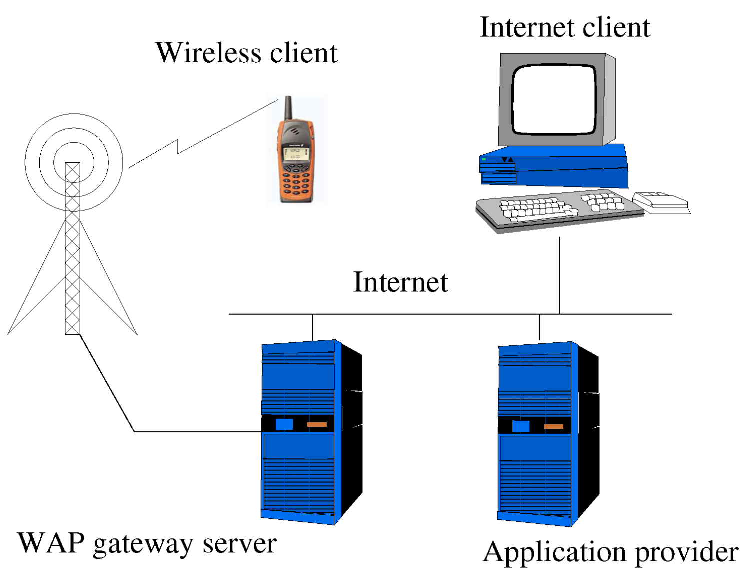

Fig. 2 The Internet network.

A. A web browser user interface

On a stationary PC or workstation, the trajectory optimization service is accessed using a standard web-browser as shown in Fig. 3. Initially, the user sets up the problem by defining aircraft configuration, initial fuel, and desired final condition. After filling in the initial form which is written in standard HTML,[14] the user sends off the form to the server for processing. The form is first processed by the server which completely sets up the trajectory optimization problem and immediately starts executing the solution of the optimization problem. After successful solution, the optimal control solution is presented to the user together with information of time, fuelburn, and distance.

In the current preliminary implementation of the Internet service, only a few different options are available. It is currently possible to compute a time or fuel optimal climb using the simplified control strategy shown in Fig. 1. For comparison, it is also possible to solve the climb problem using the original formulation as defined by Eqs. (1-4) giving a solution where the control variables are more arbitrary functions of time.

For optimal range problems, it is possible to compute a trajectory giving minimum fuel for a base to base ferry flight. The control strategy is simplified such that climb is performed at full throttle and constant calibrated airspeed, cruise at constant altitude and partial thrust, and finally descent at constant calibrated airspeed and thrust at flight idle. A number of different Swedish Air Force bases are available as user options.

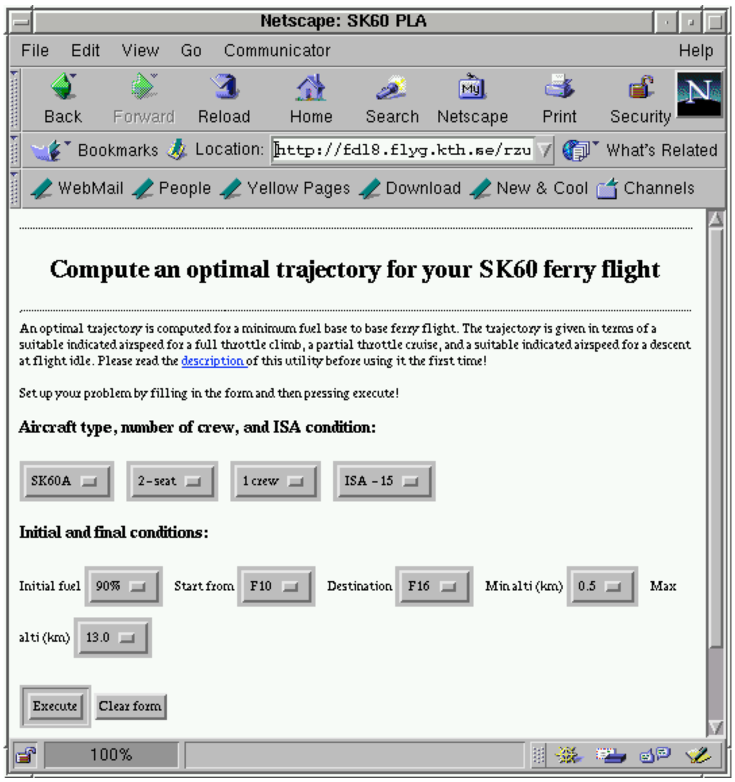

Fig. 3 The Web browser user interface.

B. The wireless application protocol

Cellular phones providing Internet access are currently becoming available in large quantities at moderate prices. The access to Internet is provided by a gateway server, see Fig. 2, which connects the Internet to the digital wireless network. The gateway server converts the information transferred on the Internet to the binary format that is used on the wireless network. The gateway server is usually maintained by the cellular phone network operator. Consequently, in order to use the trajectory optimization service from a cellular phone, the user needs a phone that supports WAP and an account with a cellular phone operator that supports the needed data services.

Implementing the support for cellular phone access in the Linux server is reasonably straightforward. The basic principle in setting up forms processing using HTML still applies although a slightly different protocol known as the Wireless Markup Language (WML)[15] is used. The WML protocol is tailored to better suit the very limited display size and restricted input devices used on cellular phones. Figure 4 shows a typical application where the pilot computes an optimal trajectory for a ferry flight before taking off.

Although solving the actual trajectory optimization problem is quite computationally intensive, this does not affect the use of the wireless Internet for user access since the computation is performed on the Linux server. However, the low bandwidth wireless connection between the server and the gateway server makes it important to minimize the amount of input data processed and sent by the cellular phone to the server and also to minimize the output results such as the optimal control strategy sent by the server to the cellular phone. The use of a simplified control strategy, as illustrated in Fig. 1, is ideal for the wireless application. The amount of input data is limited to defining the aircraft configuration and desired final condition. Likewise, the output data is very modest presenting for example the optimal calibrated airspeed and Mach number to be used for the climb together with a summary of other results such as time to climb, fuelburn, distance covered, and fuel reserve after completing the climb.

Fig. 4 Using the mobile phone user interface.

V. Numerical Examples

A few examples are presented in this section in order to illustrate the loss of optimality that is caused by the simplified control strategy. First, a minimum time to climb strategy is computed using both the simplified control strategy shown in Fig. 1 and the more general strategy obtained by using the state equations given by Eqs. (1-4). The initial conditions for the SK60 version A in two seat configuration with one pilot are level flight, initial altitude 0.1 km, V_{cal} = 350 km/h, and initial fuel 90% of maximum. The final altitude is set to 8 km.

Solving the problem using the simplified control strategy gives the trajectory shown as a solid line in Fig. 5 and the final time is t_F = 487 s. The trajectory obtained using the more general control strategy is shown as a dashed line and gives a slightly shorter time to climb at t_F = 460 s. Consequently, the loss of optimality by using the simpler control strategy is about 6% which could be significant.

Solving the same problem with the objective to achieve minimum fuelburn for the climb gives the trajectories shown in Fig. 6. Here, the simplified control strategy gives a fuelburn of 76.5 kg while the more general control strategy gives 75 kg. Consequently, the loss of optimality can be neglected completely in this case.

Finally, a range problem is considered where the minimum fuel trajectory is desired when flying a distance of 490 km. The aircraft configuration is the same as for the climb problems but the initial altitude is h = 0.5 km. The simplified control strategy involves finding a suitable calibrated airspeed for a climb at full throttle. The cruise is performed at constant altitude using partial thrust followed by descent at constant calibrated airspeed to the final altitude of h = 0.5 km.

The trajectory obtained using the simplified control strategy is shown in Fig. 7. The fuelburn for the simplified control is 256 kg in comparison to 251 kg for the more general control strategy. Consequently, the fuel savings obtained by using the more complicated control is minimal. It is interesting to see that the fuelburn and final time is so similar for the two optimal control strategies, even though the trajectories are a bit different.

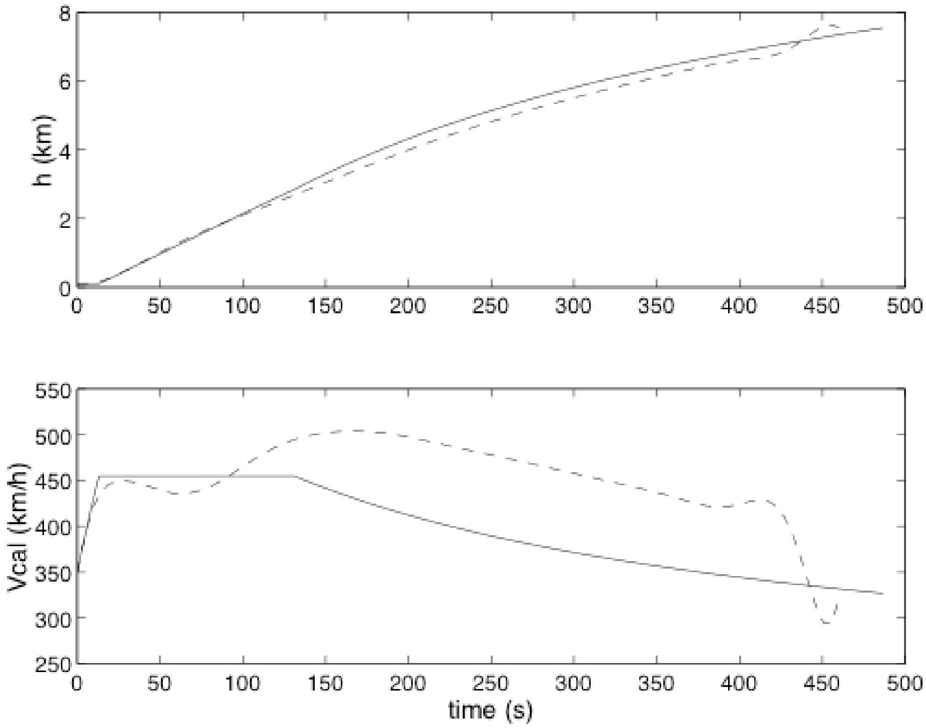

Fig. 5 Minimum time climb trajectories.

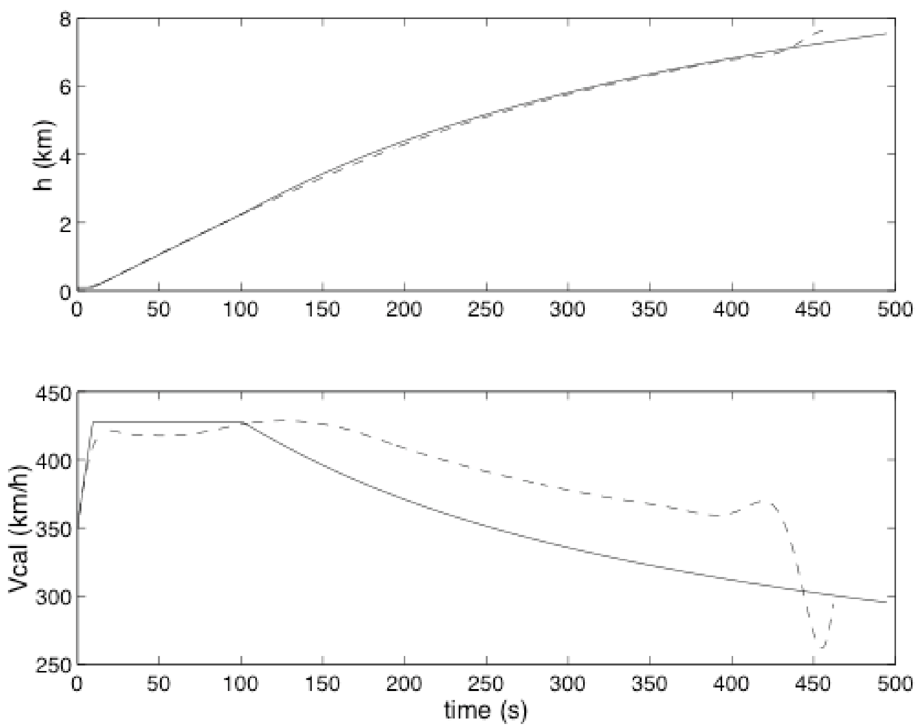

Fig. 6 Minimum fuel climb trajectories.

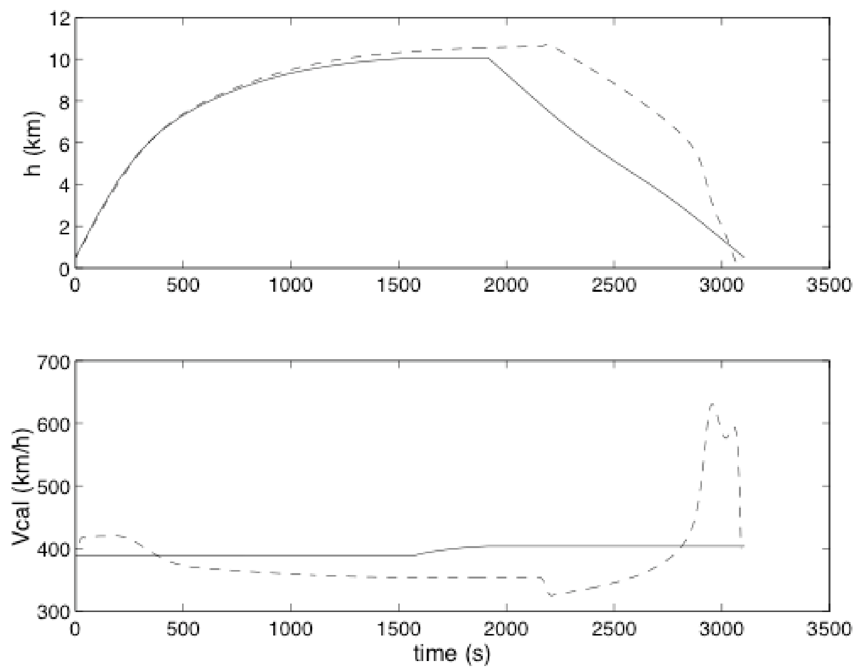

Fig. 7 Minimum fuel distance trajectories.

VI. Flight Test Results

To illustrate the accuracy of the numerical performance analysis model, a few flight test results are presented here. The particular case considered here involves finding a minimum time to climb trajectory taking the aircraft from a low speed condition at an altitude of 0.2 km to the maximum speed in level flight at an altitude of 9 km. The initial and final conditions of the trajectory optimization problem are given in Table 2. The local weather conditions were quite close to the standard atmosphere (ISA) so these conditions were assumed when computing the optimal trajectory.

Table 2 Initial and final conditions for the climb

	Initial	Final
M	0.3	0.68
h (km)	0.2	9
Fuel level	90%	-

The optimal trajectory was found by solving the optimization problem using the method described in previous sections. The optimal trajectory involves following a fairly complex scheme with changing airspeed as the altitude increases as shown in Fig. 8. The standard procedure suggested in the flight manual[9] involves initial climb at indicated air speed 400 km/h followed by a climb at constant Mach number of 0.4 until the desired altitude is reached. Acceleration to final speed is then performed in level flight. The standard procedure is also, for comparison, shown in Fig. 8.

The first flight test was performed using the standard procedure, and as can be seen in Fig. 9, the climb rate is well represented by the numerical model. The altitude as a function of time was measured using a handheld GPS recorder, no other recording device was available for these tests.

The flight tests of the optimal trajectory were performed with a co-pilot reading the instructions, essentially what airspeed to maintain at each altitude. This procedure relieves the pilot from having to read a form while simultaneously following the prescribed trajectory. Two tests were performed and recorded using the GPS unit.

The computed trajectory is shown as a dashed line in Fig. 10 and the trajectories obtained during the flight tests are shown as solid lines. As can be clearly seen from the figure, the pilot is quite successful in following the prescribed trajectory. The difference in final time is only about 4% which is small considering the many uncertainties that are present. Furthermore, the difference between the two flight tests is very small which is interesting since they were performed on two different days although with the same aircraft. The 4% difference between computation and test is by the Air Force considered to be quite close to the difference that is observed by flying the same trajectory with two different aircraft individuals. The aircraft are available in two different paint schemes, with different surface smoothness, which alone is reported to give a speed difference of about 10 km/h at maximum speed.

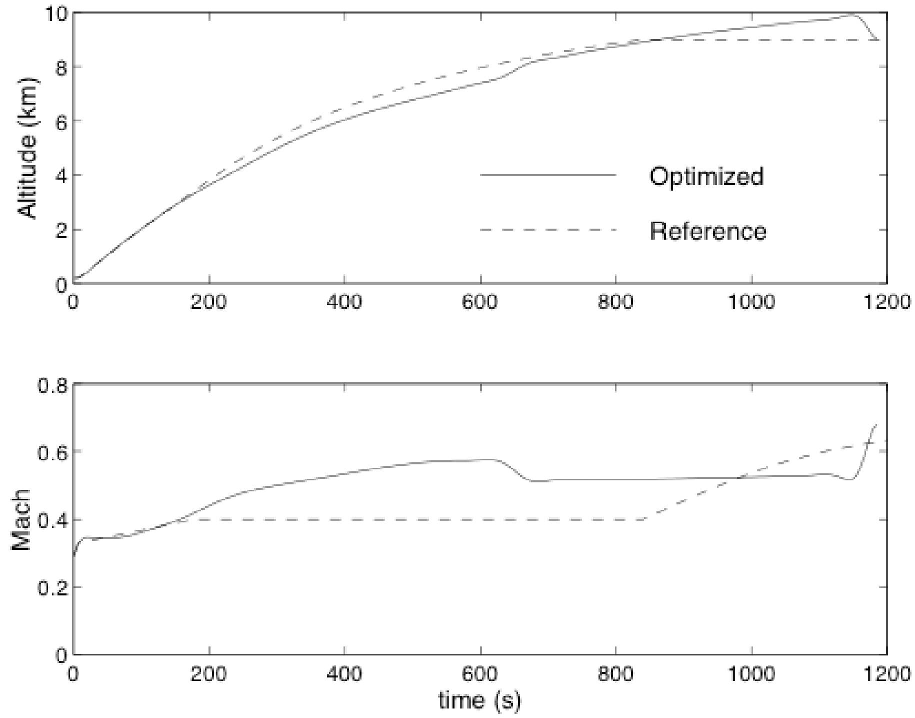

Fig. 8 The optimal and reference trajectories.

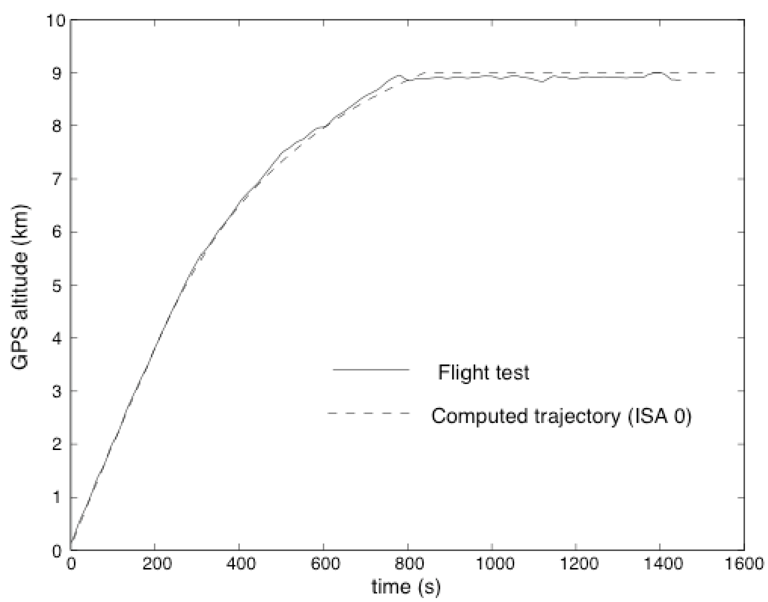

Fig. 9 The reference trajectory.

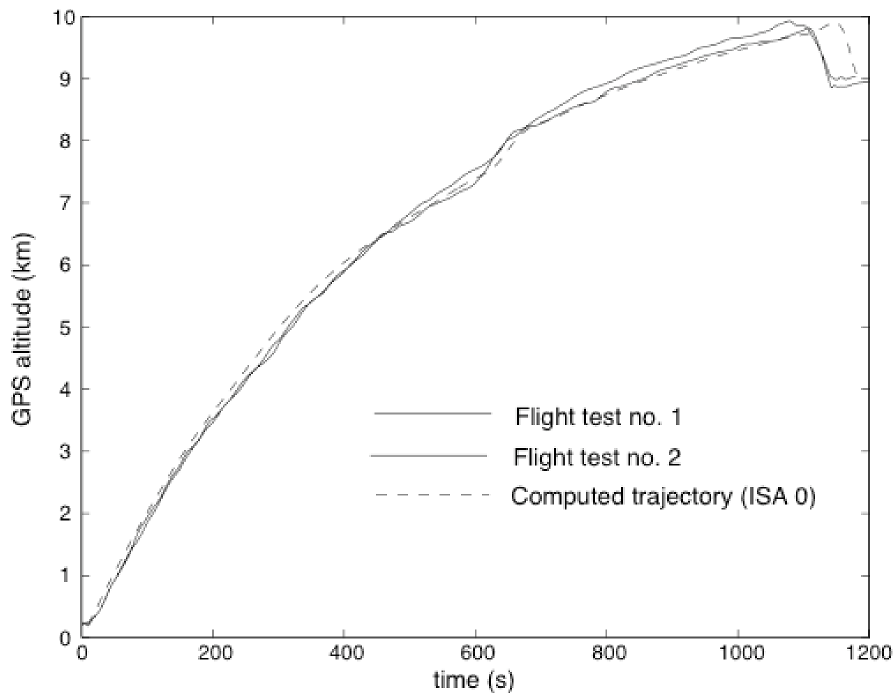

Fig. 10 The optimized trajectory.

VII. Discussion

Provided that the trajectory optimization problem is carefully modeled, computational efficiency of nonlinear optimization methods have now reached a state where interactive solution of nonlinearly constrained optimization problems in several hundred variables is possible. However, it is still impossible to guarantee that the solution will be found even when it is known to exist. The trajectory optimization problem is nonlinear and nonconvex and a numerical approximation to the solution cannot always be found. Through careful modeling of the problem and the use of methods that have been developed through many years of research and development, such as SNOPT,[12] it is possible to minimize the number of occasional failures in finding the solution. In the current implementation, failures tends to occur a few times in every 100 attempts to solve a problem in a particular class. The most common type of failure is that the solution cannot be found to the required accuracy as defined by some tolerances. Although the solution is still in most cases acceptable from a practical point of view, it is necessary to closely examine the results before accepting the solution. These cases are not easily analyzed by anyone else than the developer of the numerical methods.

To achieve full reliability in the numerical solution, it is desirable to use several different approximations to the solution. In the present implementation, a standard control strategy from the flight manual[9] is first used to compute the final time and fuelburn by integrating the state equations using the standard control strategy. This solution is then used as initial approximation to the optimization problem which further refines this initial approximation. In the cases where the optimizer fails to solve the problem, this first initial and feasible solution can be presented to the user. This way of solving a sequence of different problems in increasing detail may be the best way to provide the end user with a reliable and easy to use performance optimization tool.

In most cases, the simplified control strategy leads to more conservative results, for example higher fuel consumption. This is to be expected since the simplified control strategy can be seen as adding more constraints to the optimization problem which reduces the set of feasible solutions and this can of course never lead to a better solution in terms of objective function value at the optimum. Further, it is important to point out that the differences between using different control strategies can be significant. A too simple control strategy can make the solution much worse than a more general control strategy. This is particularly true for supersonic minimum time to climb problems where a more complex control strategy may give significant reductions in time as compared to a simplified strategy involving climb at constant speed and accelerations at constant altitude. The pilots flying the SAAB 105 (SK60) wanted these forms of simplified control strategy to reduce workload at the expense of some reduction in optimality.

The use of Internet for providing distributed access has proven successful. The user interface is familiar to most people, and it is also very convenient to provide manual pages in the same format. This way, there is no need to remotely install software and to provide printed documentation saving significant costs. Although there are still some significant bottlenecks in the use of wireless Internet services on a cellular phone, mainly because of the strong expansion in the number of users, this technique offers very interesting possibilities for field use. The cellular phone is useful right on the pad so that the pilot can make the analysis just before take-off. Installing the capability in the aircraft is possible, but the certification process for installed hardware in the aircraft, communications network and software is a daunting task.

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References

[1]Hargraves, C. R., and Paris, S. W., "Direct Trajectory Optimization Using Nonlinear Programming and Collocation," *Journal of Guidance, Control, and Dynamics*, Vol. 10, 1987, pp. 338–342.

[2]Betts, J., "Experience with a Sparse Nonlinear Programming Algorithm," *Large-scale Optimization with Applications*, Vol. 93, edited by A. Conn and F. Santosa, Institute for Mathematics and its Applications, Springer, 1997, pp. 53–72.

[3]Shi, Y. Y., Nelson, R., Young, D. H., Gill, P. E., Murray, W., and Saunders, M. A., "The Application of Nonlinear Programming and Collocation to Optimal Aeroassited Orbital Transfer Trajectory," AIAA Paper 92-0734, Jan. 1992.

[4]Gill, P. E., Murray, W., and Saunders, M. A., "Large-scale SQP Methods and Their Application in Trajectory Optimization," Report NA 93-3, Dept. of Mathematics, Univ. of California San Diego, 1993.

[5]Ringertz, U. T., "Flight Testing an Optimal Trajectory for the Saab J35 Draken," *Journal of Aircraft*, Vol. 37, No. 1, 2000, pp. 187–189.

[6]Ringertz, U. T., "Multistage Trajectory Optimization Using Large-Scale Nonlinear Programming," TR 99-25, Dept. of Aeronautics, Royal Institute of Technology, 1999.

[7]Ringertz, U. T., "Optimal Trajectories in Three Dimensions," TR 99-30, Dept. of Aeronautics, Royal Institute of Technology, 1999.

[8]Taylor, M. H. A. M., *Understanding WAP*, Artech House Publishers, 2000.

[9]Försvarets Materielverk, Speciell f¨orarinstruktion fpl SK60, (SFI:IV), 1997.

[10]Anderson, J. D., *Introduction to Flight*, Aerospace Science Series, McGraw-Hill, 1989.

[11]Brenan, K. E., "Differential-Algebraic Equations Issues in the Direct Transcription of Path Constrained Optimal Control Problems," Aerospace Report ATR-94(8489)-1, The Aerospace Corporation, 1993.

[12]Gill, P. E., Murray, W., and Saunders, M. A., "User's Guide for SNOPT 5.3: A FORTRAN Package for Large-Scale Nonlinear Programming," Report NA 97-X, Dept. of Mathematics, Univ. of California San Diego, 1997.

[13]Ringertz, U. T., "An Optimal Trajectory for a Minimum Fuel Turn," *Journal of Aircraft*, Vol. 37, No. 5, 2000, pp. 932–934.

[14]Kennedy, C. M. A. B., *HTML The Definite Guide*, O'Reilly & Associates, 1998.

[15]*WML Reference*, Nokia, Finland, 2000.